An Account Concerning the Resolution of Equations in Numbers; imparted by Mr. Iohn Collins.

This Account should have been annex'd to what was discoursed of Monsieur *Slusius* his *Mesolabe* in the precedent Tract, if then we had found room for it. For, the Reader having there understood, how farr the *Geometrick* part of *Algebra* is advanc'd by that excellent person, 'twas likely, he would be inquisitive to hear somewhat concerning the *Exegesis Numerosa*, or the Resolution of Æquations in *Numbers*. For whose satisfaction herein, we shall here insert the Account then omitted, being part of a narrative, formerly made by M. *Iohn Collins* touching some late Improvements of Algebra in *England*, upon the occasion of its being alledged, that none at all were made since *Des Cartes*.

1. It hath been observ'd by divers of this Nation, that in any Æquation, howsoever affected, if you give a Root, and find the Absolute number or Resolvend (which *Vieta* calls *Homogeneum Comparationis*) and again give more Roots and find more Resolvends, that if these Roots or rather rank of Roots be assum'd in *Arithmetical* progression, the Resolvends, as to their first, second or third differences, &c. imitate the Laws of the pure Powers of an Arithmetical progression of the same degree, that the higest Power or first term of the Equation is of. e. g. In this Equation $aaa - 3aa + 4a = N$,

If *a* be =		Then N. or the Absolutes or Resolvends will be found to be	1. *dif.*	2. *dif.*	3. *dif.*
10		740			
			218		
9		522		48	
			170		6
8		352		42	
			128		6
7		224		36	
			92		
6		132			

To wit the 3d. differences of those *Absolutes* are equal, as, in the Cubes of an Arithmetical Progression.

2. To find, what habitude those *differences* have to the *Coefficient*, of the Equation, 'is best to begin from an Unit.

3. In any Arithmetical Progression, if you multiply Num-

bers by *pairs*, you shall create a rank of Numbers whose 2*d*. differences are equal; and if by *ternaries*, then the 3*d*. differences of those Products shall be equal. And how to find the greatest Product of an Arithmetical Progression of any number of terms having any common difference assign'd, contain'd in any Number proposed, is shew'd by *Pascal* in his Tract *du Triangle Arithmetique*, where he apply's it to the Extraction of the Roots of simple powers.

4. It appears, How this rank may be caried easily by Addition, till you have a *Resolvend* either equall or greater or lesse, than that proposed.

5. When you have a *Majus* and *Minus*, you may interpole as many more termes in the Arithmetical Progression as you will, that is to say, Subdivide the Common difference in the Arithmetical Progression, and render it lesse; and then renew, and find the Resolvends, which are easily obtain'd out of the Powers and their Coefficients, which are suppos'd knowne, and may be readily rais'd from a *Table* of *Squares* and *Cubes*, &c. with which kind the *Reader* may be furnisht in *Guldini Centrobaryca* and *Babingtons Fireworks*: By this means you may obtain divers Figures of the Root; and then the General Method of *Vieta* and *Harriot* runs away more easily, and is so far improv'd, that after any figure is plac'd in the Root, most certain Characters are given to know by aide of the subsequent *Dividend* and *Divisor*, Whether the figure before assum'd be too great or too small: or lastly it may well be concluded, that, as in Logarithmes, when you propose such an one as is not absolutely given in the *Canon*, you doe by Proportional Work, using the aid of their first differences (when their Absolute Numbers differ by *Unit*) find the absolute Number true to 5. or 6. places further than the *Canon* gives it (the reason whereof is, that the first Differences doe likewise agree to about the same Number of places;) that I say, the like may be done in Æquations, after divers of the first figures of the root are found; provided there be the like agreement in the first differences of the interpoled Resolvends.

Moreover we ought here to take notice of a more subtile kind of Interpolation, common to all gradual Ranks or Progressions of Numbers, wherein Differences happen to be equal: Of which

kind

kind the *Reader* may find Examples in *Briggii Arithmetica Logarithmica et Trigonometria Britannica*, relating to Logarithmes, Sines, and the Powers of an Arithmetical Progression: But the method there deliver'd may be rendred more easy and general, *viz.* by aid of a *Table* of figurat Numbers, by deriving Generating differences *sought*, from those *given*; a doctrine, that easily flows from *Mercators Logarithmotechnia*, and of use in the Case in hand, should we suppose these Powers and their Coefficients vnknown, or a *Table* of *Squares* and *Cubes* wanting, and give nothing more, than a few *Resolvends* belonging to equal Moments or Spaces. And this may likewise be of good use in *Guaging*, when having the Contents of a Solid, for every 3. Inches, more or lesse, given, without knowing the dimensions of the Figure, and even in most Cases, when the differences are Progressive of one kind, without knowing the Figure it self, having nothing given but its Contents at several equal Parallel distances, each such distance may be subdivided, and made as many as you please, and the respective Contents found by this general Method of Interpolation.*

After one Root is obtained, the Methods of *Huddenius* and others will depresse the Equations so as to obtain more, and consequently all of them.

6. It is easy, by a *Table* of figurate Numbers to give the sum of any such Rank or any term in it relating to a known part of the *Series* of Equals or Roots; but *è converso*, giving the *Resolvend* to find the Root, coms to an Equation as difficult as that proposed; as in D. *Wallis* his Chapter of Figurate Numbers.

*Nota. *The Author (M.* Collins) *haveing explain'd* Mercators Logarithmotechnia *in English, and illustrated the elegant Doctrine thereof with Examples, hath likewise handled this work of Interpolation, and makes the* Logarithmes *true to 25. or 30. places of figures by meer Division (or Proportion;) having herein advanc'd that Author's doctrine thereof by Division, which (as 'tis there illustrated) did not* seem *to extend farr enough. This hath already been communicated, some Months since, to some of the Members of the* R. Society, *and may be expected hereafter.*

7. Some affirm, they can give good Approaches for the obtaining a Root of any *pure* power, *affected* Equation, or for the finding of any of the mean Proportionals in any Rank between two extreams given.

8. Others pretend to have found out a method (incited thereto by an example *in Albert Gerards Invention Nouuelle en Algebre à Amsterdam 1629.*) so much, by comparing of Equations, to increase

crease or diminish the unknown Root of Equation, as to render it a whole number (or lesse differing therefrom, than any Error assign'd,) and by *Albert Gerards* Method of *Aliquot parts* to find the same, and thereby the Root sought, although it be a Mixt Number, Fraction, or Surd.

Probably this may sympathise with what is promised by the Learned *Huddenius in Annexis Geometriæ Cartesianæ*, where he saith, he intended not *then* to publish certain Rules, he had ready; whereof one was, To find out all the irrational Roots both of Literal and Numeral Equations. This must be understood when such Roots are *possible*; for 'tis certain, there are infinite Equations, whose Roots are no ways explicable, either in whole or mixt numbers, Fractions or Surds, and can be no otherwise explain'd, but by a *quàm proxime*.

9. The Author of this Narrative considering, that the *Conick Sections* may be projected from lesser Circles placed on the Sphere, and thence easily (otherwise than hitherto hath been handled) described by Points, and that by their Intersections some Spherick Problem is determined, accordingly he found, that this following Problem according to the various Scituation of the Eye, and of the Projecting Plain, would take in all Cases.

The Distances of an unknown Star are given from two Stars of known Declination and Right Ascension; the Declination and Right Ascension of the unknown Star is required.

And saith, he hath observed, that, admitting the Mechanisme of dividing the Periphery of a Circle into any number of equal parts, or (which is equivalent) the Use of a Line of Chords, that this Problem, wherever the Eye be plac'd, may be resolved by Plain Geometry, and yet the Ey shall be so plac'd, as to determin it by the Intersections of the Conick Sections; consequently those Points of Intersection (the Species and Position of the figures being given) may be found without describing any more Points than those sought, and the Lengths of Ordinates falling from thence on the *Axes* of either figure calculated by mixt Trigonometry, and hence likewise the Roots of all *Cubick* and *Bi-quadratick* Equations found by *Trigonometry*.

For giving from the *Mesolabe* mention'd the *Scheme* that finds these Roots, it will then be required to fit those *Sections* into *Cones*, which have their *Vertex* either in the Center; or an assigned point in the Surface of the Sphere, to which they

relate

relate as projected, and proceed to the resolution of the Problem propos'd: And how to fit in those Sections, see the 7. books of *Apollonius*, *Mydorgius*, the 3d. Volume of Des *Cartes's* Letters, *Leotaudi Geometrica practica*, *Andersonii Exercitat. Geometricæ.*

As to the Problem it self, it is determin'd on the Sphere by the Intersections of the two lesser Circles of Distance, whose *Poles* are the known Starrs. And this Problem hath divers *Geometrick* ways of resolution.

1. By *Plain Geometry* (in the sense before-mentioned;) Supposing a Plain to touch the Sphere at the *North-pole*: if the Eye be at the *South-pole*, projecting those Circles into the said Plain, they are still Circles (by reason of the sub-contrary Sections of the Visual Cones) whose Centers fall in the sides of the Right-lin'd Angle, made by the Projected *Meridians*, that pass through the known Starrs; and thus the Problem is easily solv'd in this manner.

2. If it be required to be performed by *Conick Geometry*; in one case it may be done, by placing the Ey at the Center of the Sphere, and projecting as before; to wit, when the longer *Axes* of the figures being produced concur above the *Vertex*; Here the *Problem* is determined by the Intersections of two Conick Sections (whereof a Circle cannot be one, unless its Center be in the *Axis* of the other figure.) And in this second Case these points of Intersection fall in the same right line or projected *Meridian*, they did before, but at a more remote distance from the Pole-point, to wit, in the former Supposition, the Solar distance was measur'd by a Right line, that was the double Tangent of half the Arch; here it is the Tangent of the whole Arch. Hence it is evident, how one *Projection* may beget another, yea infinite others, altering the Scale; and how the lesser Circles in the *Stereographick* Projection help to describe the Conick Sections in the *Gnomonick* Projection: But (to reduce the matter to one common *radius*) if we suppose two Spheres equal, and so placed about the same *Axis*, that the Pole-point of the one shall pass through the Center of the other, and the Touch-plain to pass through the said Center or Pole-point, and that a lesser Circle hath the same position in the one as in the other; Then, if the Ey be at the South-Pole of the one, it is at the Center of the other; and any projected *Meridian* drawn from the projected Pole-point to pass through both the projections of these les-

ser Circles, the distances of the Points of intersection are the Tangents of the half and the whole Arch of the Meridian so intersected. But as to the Points of Intersection, which determine the Problem proposed, they may be found without the aid of the former way, from a *Gnoonmick* and *Stereographick* method of measuring and setting off the sides and angles of Spherical Triangles in those Projections, which is necessary in what follows.

3. If the Problem is to be perform'd by *Mixt Geometry*, as by a Circle and either a *Parabola*, *Hyperbola*, or *Ellipsis*, the Circle may be conceived to be the Sub-contrary Section of a Cone projected by the Eye at the *South-pole*, and any of the rest of the Sections by the Eye at the Center of the Sphere.

4. If by any of the *Conick Sections* however posited; the projecting Plain may remain the same, but the Eye must be in some other part of the *Surface* of the Sphere, and not in the *Axis*.

These things were mention'd to invite the Learned to their Consideration: I shall only further adde, that we cannot say, what may be expected from the labours and endeavors of divers Learned men of this Nation, particularly from Dr. *Wallis*, who hath so excellently resolved and constructed all *Cubick Æquations* at the end of the first Treatise of his *Opera Mathematica* by aide of a Cubick *Parabolaster*, mentioning, that by such Curves the Roots of all Æquations may be found: And who hath promised a Treatise of *Algebra* and *Angular Sections*, wherein the Reader need not doubt to meet with satisfaction in these Mysteries. Nor ought we to omit the mentioning of the Modest and Learned Mr. *Barrow*, who (among many other excellent Subjects, and particularly his Opticks now, at the Press) hath perform'd, what the famous Italian Geometer *Mich. A. Ricci* hath promis'd in *Exercitat. Geometrica* (printed at *Rome* 1666. and lately reprinted here) about *Curves* of several degrees, that serve to determine and resolve all *Æquations*: which hath likewise been done by other Learn'd men of this Nation.

An Account of Books.

I. PRÆLUDIA BOTANICA Roberti Morison *Scoti Aberdonensis*. Londini, impensis *Jac. Allestry*, 1669. in 8°.

This Prelude of this Excellent Botanist hath two parts; The first gives us an Alphabetical Catalogue of all the Plants in the Royal